LECTURE NOTES: INCENTIVES AND MORAL HAZARD

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1. A model of incentives and *Moral Hazard*

A **venture capitalist** (*she*) is considering investing in a project that involves an **entrepreneur** (*they*) developing a product. While the venture capitalist has the financial resources to fund the project, she lacks the technical knowledge to develop the product herself. On the other hand, the entrepreneur has the expertise to develop the product but lacks the necessary funding. The key question is: how should the venture capitalist structure the funding offer to the entrepreneur?

The entrepreneur exhibits some degree of risk aversion. Specifically, for simplicity, we assume that the entrepreneur's utility for an amount x (in pounds) is given by $\sqrt{x}$. In contrast, the venture capitalist is well-diversified and therefore exhibits no risk aversion, meaning her utility for x pounds is simply x.

It is well known that the project involves developing a product that will have market demand. However, the final quality of the product depends on the effort exerted by the entrepreneur (high or low) and on the state of nature (good or bad) that is only observable by the entrepreneur. If the entrepreneur exerts high effort and the state is good, denoted by $\overline{e}$, then the final product will be of *superb* quality with probability p and of *OK* quality with probability $1-p$. Conversely, if the state is good and entrepreneur exerts low effort, denoted by $\underline{e}$, then the probability of achieving a superb product drops to q, with the probability of an OK-quality product being $1-q$. If the state is bad, then the product will never be of superb quality and is of *OK* quality with probability 1. The venture capitalist holds beliefs about the state of nature: the state is good with probability $\eta \in (0,1)$.

Since $p > q$, high effort increases the likelihood of a superb product, only in the good state. However, exerting high effort comes at a higher cost to the entrepreneur than

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exerting low effort. Specifically, the cost of effort is given by:

$$C(\overline{e}) = \overline{C} > \underline{C} = C(\underline{e}) > 0.$$

We assume that the cost of effort enters linearly into the entrepreneur's preferences. Furthermore, high effort is assumed to be worthwhile in the sense that the additional profit from a superb product, $\overline{\pi}$, more than compensates for the extra effort cost:

$$\boxed{\sqrt{\overline{\pi}} - \sqrt{\underline{\pi}} > \overline{C} - \underline{C}.}$$

Here, $\underline{\pi} > 0$ represents the profit generated by a product of OK quality.

To complicate matters, the entrepreneur has an alternative opportunity: they could work as an employee in the private sector, earning a utility of $\underline{U}$ by quitting the job at any point. At first glance, this should not be a major concern, as even producing a product of OK quality is preferable to working in the private sector:

$$\boxed{\sqrt{\underline{\pi}} - \underline{C} \geq \underline{U} \geq \overline{C}.}$$

Finally it is well known that the expected return of high effort, net of the cost of high effort and the opportunity cost, not only is positive,

$$\boxed{p\overline{\pi} + (1-p)\underline{\pi} > \underline{U} + \overline{C}} \tag{1}$$

but it is also higher than the expected return of low effort net of the cost of low effort and the opportunity cost

$$q\overline{\pi} + (1-q)\underline{\pi} - \underline{U} - \underline{C}$$

which is equivalent to the following condition

$$\boxed{(p-q)(\overline{\pi} - \underline{\pi}) > \overline{C} - \underline{C}} \tag{2}$$

and we assume to hold in these notes. Furthermore, we assume, for technical reasons, that the preferences of the entrepreneur are well defined and verify

$$\boxed{p(\underline{U} + \underline{C}) \geq q(\underline{U} + \overline{C})} \tag{3}$$

Assumption 1. *Conditions* (1), (2), *and* (3), *hold.*

Furthermore, we are going to assume that high effort is preferred to low effort if effort were observable

Assumption 2. *The project is worthwhile to be undertaken in the good state if effort were observable:*

$$p\overline{\pi} + (1-p)\underline{\pi} - (\underline{U} + \overline{C})^2 > 0$$

and high effort in state G is optimal

$$p\overline{\pi} + (1-p)\underline{\pi} - (\underline{U} + \overline{C})^2 > q\overline{\pi} + (1-q)\underline{\pi} - (\underline{U} + \underline{C})^2$$

1.1. **The wage structure as a game.** This is a strategic situation between two players that can be analyzed as a game of incomplete information. We are going to assume the following fundamental structure and sequence of actions:

(1) The venture capitalist is the first to move an offers a contract to the entrepreneur. The only observable variable for the venture capitalist is the output: she does not observe the state of nature nor the effort exerted by the entrepreneur.
   (a) The contract that the venture capitalist offers to the entrepreneur comes in two parts: a payment associated with the outcome being of OK quality which we denote by w_{OK} and a payment associated with the outcome being of superb quality which we denote by w_S.
   (b) We will assume that these payments cannot be negative $w_{OK} \geq 0$ and $w_S \geq 0$. This assumption is usually referred as limited liability: the venture capitalist cannot charge the entrepreneur for participating in the project.

(2) The entrepreneur moves second after observing the wage structure offer by the venture capitalist.
   (a) He can either accept the contract or take the outside option $\underline{U}$.
   (b) If she takes the outside option the game ends and the venture capitalist collects 0 and the entrepreneur receives a payoff of $\underline{U}$. Again, this is an assumption as we could have assumed that drafting the contract is costly.
   (c) If the entrepreneur accepts the contract, the game moves onto the next stage.

(3) Once the contract has been accepted, the entrepreneur observe the state of nature (Nature moves).[1]

[1]This comment is advanced: if the entrepreneur observes the state of nature, before accepting the contract or not the game changes significantly. In particular, it potentially introduces an adverse selection effect which requires to expanding the game appropriately. In particular, the venture capitalist could offer two different contracts to screen the type of information observed by the entrepreneur. This is an advanced

(4) The entrepreneur decides whether to exert high or low effort.

(5) Nature moves and determines the outcome of the game.

2. A benchmark: Imperfect but symmetric information

If **VC** were able to observe the effort of the entrepreneur and the state of nature, then she would make the entrepreneur a take it or leave it offer of a fixed wage of $w^*(G) = (\underline{U} + \overline{C})^2$ in the good state if the effort is high and 0 otherwise, and an offer of $w^*(B) = (\underline{U})^2$. This is simply due to the following two facts. First, in the bad state good effort does not payoff (the outcome is always an OK product) and in the good state it is worthwhile to make the effort. Second, in both cases the entrepreneur needs to be compensated for not taking the outside option.

We can describe the game with complete information but imperfect in the following way.

- There are three players: **VC** (venture capitalist), **E** (entrepreneur) and **N** (Nature)

- There are a lot of terminal histories but all of them have one element of each of the following sets

$$\{\{G,B\} \times \{\underline{w},\overline{w}\} \times \{N,Y\} \times \{\underline{e},\overline{e}\} \times \{\underline{\pi},\overline{\pi}\}\}$$

  where $\{\underline{w},\overline{w}\}$ is the wage structure offer by the by **VC** and $\{0,1\}$ is **E**'s decision to accept or reject the offer.

- The Player Function F

$$\begin{aligned} F(\emptyset) &= \mathbf{N} \\ F(G,B) &= \mathbf{VC} \\ F(\{G,B\},\{\underline{w},\overline{w}\}) &= \mathbf{E} \\ F(\{G,B\},\{\underline{w},\overline{w}\},\{N,Y\}) &= \mathbf{E} \\ F(\{G,B\},\{\underline{w},\overline{w}\},\{N,Y\},\{\underline{e},\overline{e}\}) &= \mathbf{N} \end{aligned}$$

  – At this point one can observe the strategies of each player

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topic to which some is seen in next years Advanced Microeconomics and Incomplete contracts options and belongs to the area of optimal design of contracts of personnel economics.

∗ **VC** can choose two pairs of numbers in the real line conditional on the state:[2]

$$[(\underline{w}(G), \overline{w}(G)), (\underline{w}(B), \overline{w}(B))] \in [0, \infty)^2$$

∗ **E** can choose for each offer in each state to accept or not the contract and if accepted, then the effort they will make. That is, for each state $s \in) G, B),$

$$\{(Y(\underline{w}, \overline{w}, s), e(\underline{w}, \overline{w}, s), N\} \quad \text{where } e(\underline{w}, \overline{w}) \in \{\underline{e}, \overline{e})\}$$

∗ Player **N** plays mechanically in the end in the following way:

$$1 - \Pr(\underline{\pi}|\overline{e}, G) = \Pr(\overline{\pi}|\overline{e}, G) = p > q = \Pr(\overline{\pi}|\underline{e}, G) = 1 - \Pr(\underline{\pi}|\underline{e}, G)$$

$$\Pr(\underline{\pi}|\overline{e}, B) = \Pr(\underline{\pi}|\underline{e}, B) = 1 > 0 = \Pr(\overline{\pi}|\overline{e}, B) = \Pr(\overline{\pi}|\underline{e}, B)$$

- Information for each player (subhistories)

$$I_{\mathbf{VC}} = (\{G, B\}) \qquad I^1_{\mathbf{E}} = (\{G, B\}, \{\underline{w}, \overline{w}\}) \qquad I^2_{\mathbf{E}} = (\{G, B\}, \{\underline{w}, \overline{w}\}, Y)$$

- The payoffs

$$U_j(s, \{\underline{w}, \overline{w}\}, (Y, \underline{e}), \pi) = \begin{cases} \sqrt{\underline{w}} - \underline{C} & j = E, s \in \{G, B\},\ \pi \in \{\underline{\pi}, \overline{\pi}\} \\ \underline{\pi} - \underline{w} & j = VC, s \in \{G, B\},\ \pi = \underline{\pi} \\ \overline{\pi} - \underline{w} & j = VC, s \in \{G, B\},\ \pi = \overline{\pi} \end{cases}$$

$$U_j(s, \{\underline{w}, \overline{w}\}, (Y, \overline{e}), \pi) = \begin{cases} \sqrt{\overline{w}} - \overline{C} & j = E, s \in \{G, B\},\ \pi \in \{\underline{\pi}, \overline{\pi}\} \\ \underline{\pi} - \overline{w} & j = VC, s \in \{G, B\},\ \pi = \underline{\pi} \\ \overline{\pi} - \overline{w} & j = VC, s \in \{G, B\},\ \pi = \overline{\pi} \end{cases}$$

$$U_j(s, \{\underline{w}, \overline{w}\}, N) = \begin{cases} \underline{U} & j = E, s \in \{G, B\}, \overline{\pi}\} \\ 0 & j = VC, s \in \{G, B\},\ \pi = \underline{\pi} \end{cases}$$

where we used $\{\underline{w}, \overline{w}\}$ to simbolize that this is a pair of two numbers in the real line.

We can also describe the game using a Game tree which we do in two different figure: in Figure 1 we show the game tree that follows after nature has played state G, which

[2]Note that the salaries cannot be negative by assumption.

occurs with probability η and in Figure 2 we show the game tree that follows after nature has played state B, which occurs with probability $1-\eta$.

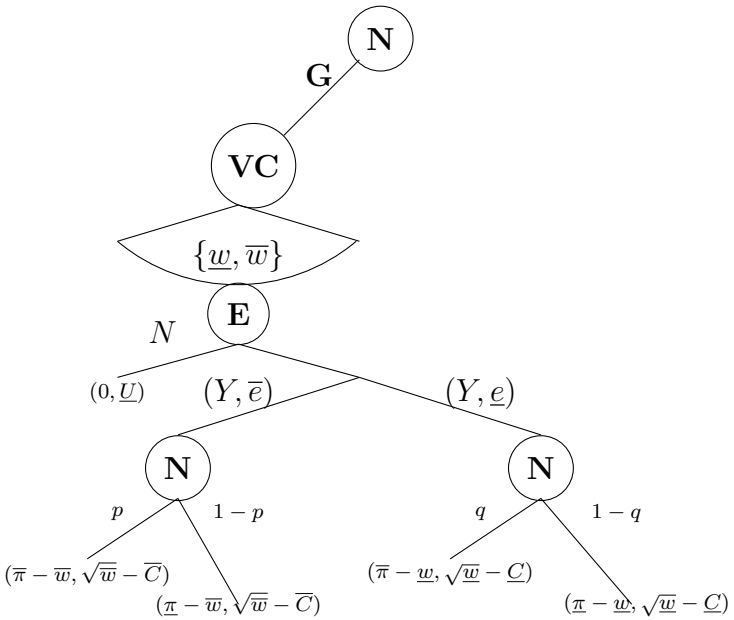

FIGURE 1. Game tree describing the game between the venture capitalist (**VC**) and the entrepreneur (**E**) under symmetric but imperfect information following state G. The arc signifies that there are multiple possible offers $\{w_{OK}, w_S\}$; in particular $\{w_{OK}, w_S\} \in [0, \infty)^2$. We have separated the decision of **E** in two different nodes for convenience only.

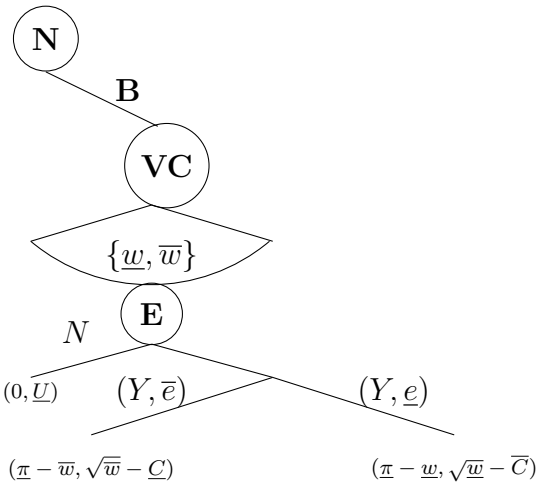

FIGURE 2. Game tree describing the game between the venture capitalist (**VC**) and the entrepreneur (**E**) under symmetric but imperfect information following state B. The arc signifies that there are multiple possible offers $\{w_{OK}, w_S\}$; in particular $\{w_{OK}, w_S\} \in [0, \infty)^2$. We have separated the decision of **E** in two different nodes for convenience only.

Although sequential games have both Nash equiibria and Subgame Perfect Equilibria, we focus here on the latter. The next proposition describes the Unique Subgame Perfect Nash equilibria[3]

Proposition 1. ***VC*** *offers* $(\overline{w}, \underline{w})$ *such that*

$$\underline{w}(B) = (\underline{U})^2 \geq \overline{w}(B) - \overline{C} \quad \textit{and} \quad \overline{w}(G) = (\underline{U})^2 \geq \underline{w}(G) - \underline{C}$$

and ***E*** *uses in state* B

$$BR(\overline{w}, \underline{w}) = \begin{cases} (Y, \overline{e}) & \textit{if } \overline{w} - \overline{C} > \max\{\underline{w} - \underline{C}, \underline{U}\} \\ (Y, \underline{e}) & \textit{if } \underline{w} - \underline{C} \geq \max\{\overline{w} - \overline{C}, \underline{U}\} \\ N & \textit{if } \underline{U} > \max\{\underline{w} - \underline{C}, \overline{w} - \overline{C}\} \end{cases}$$

and in state G

$$BR(\overline{w}, \underline{w}) = \begin{cases} (Y, \overline{e}) & \textit{if } \overline{w} - \overline{C} \geq \max\{\underline{w} - \underline{C}, \underline{U}\} \\ (Y, \underline{e}) & \textit{if } \underline{w} - \underline{C} > \max\{\overline{w} - \overline{C}, \underline{U}\} \\ N & \textit{if } \underline{U} > \max\{\underline{w} - \underline{C}, \overline{w} - \overline{C}\} \end{cases}$$

with payoffs

$$U_j(s) = \begin{cases} p\overline{\pi} + (1-p)\underline{\pi} - (\underline{U})^2 & \textit{if } j = \boldsymbol{VC}, s = G \\ \underline{\pi} - (\underline{U})^2 & \textit{if } j = \boldsymbol{VC}, s = B \\ (\underline{U})^2 & \textit{if } j = \boldsymbol{E}, s = G, B \end{cases}$$

and the agent exert high effort in the good state and low effort in the bad state.

3. An introduction to moral hazard

Moral hazard arises when one party in a transaction can take actions that affect the other party's outcomes, but those actions are hidden or difficult to observe. Classic examples include insured individuals who behave more recklessly because they do not bear the full costs of their actions, or managers who pursue personal benefits at the expense of shareholders. This "hidden action" problem creates a misalignment of incentives that can undermine efficient outcomes.

[3]Checking that this is indeed correct is a left as an exercise in applying backward induction.

From a game theory perspective, moral hazard is typically modeled as a principal–agent problem with asymmetric information. The principal (e.g., insurer or employer, or **VC** in our model) cannot perfectly monitor the agent (e.g., insured individual or employee, or **E** in our model), which leads to actions and results that deviate from the principal's best interests. Designing contracts or incentives to mitigate this hidden action problem becomes crucial in these scenarios. This will be the focus of the next section.

3.1. **Moral Hazard in the VC-E relationship.** To understand the problem of moral hazard it is a good idea to start thinking first as if **E** were the actual owner of the company and had the resources to develop the product and then compare the result with a situation in which **VC** is the owner and employes **E** and pays a fixed salary without any performance pay. By definition, **E** can observe their own action and assuming that they observe the state we have the following simple decision rule in the first case:

Proposition 2. *If **E** owns the company and develops the product themselves, and Assumption 2 holds, they exerts no effort in state B and high effort in state G. The expected payoff given this rule, before the state is realized, is given by*

$$\mathbb{E}U^*_{\boldsymbol{E}} = \eta\left(p\sqrt{\overline{\pi}} + (1-p)\sqrt{\underline{\pi}} - \overline{C}\right) + (1-\eta)\left(\sqrt{\underline{\pi}} - \underline{C}\right)$$

We assume further that the product is worth developed if **E** would be able to do so

Assumption 1. *The project is worth pursuing by the entrepreneur $\mathbb{E}U^*_{\boldsymbol{E}} > \underline{U}$:*

$$\eta p\sqrt{\overline{\pi}} + (1-\eta p)\sqrt{\underline{\pi}} - \eta\overline{C} - (1-\eta)\underline{C} \geq \underline{U}$$

When **VC** owns the company but **E** develops the product, the choice of the non-performance pay unique wage is the outcome of a a game. To analyze this game we need to first describe it properly with the new information set up; i.e. **VC** does not observe anything and **E** observes the state *after* accepting or declining the contract.[4] We are going to describe it assuming that **VC** can use a performance-pay wage schedule, $w_{OK} \neq w_S$ and later on see what happens when we force her to pay a salary $w^* = w_{OK} = w_S$.

The game us described in extensive form as follows:

- There are three players: **VC** (venture capitalist), **E** (entrepreneur) and **N** (Nature)

[4]The timing here is important, as changing the order when **E** observes the state also generates a problem of adverse selection and screening.

- There are a lot of terminal histories but all of them have one element of each of the following sets

$$\{\{w_{OK}, w_S\} \times \{N, Y\} \times \{G, B\} \times \{\underline{e}, \overline{e}\} \times \{\underline{\pi}, \overline{\pi}\}\}$$

where $\{w_{OK}, w_S\}$ is the wage structure offer by the by **VC** and $\{N, Y\}$ is **E**'s decision to accept or reject the offer.

- The Player Function F

$$\begin{aligned} F(\emptyset) &= \mathbf{VC} \\ F(\{w_{OK}, w_S\}) &= \mathbf{E} \\ F(\{w_{OK}, w_S\}, \{N, Y\}) &= \mathbf{N} \\ F(\{w_{OK}, w_S\}, \{N, Y\}, \{G, B\}) &= \mathbf{E} \\ F(\{w_{OK}, w_S\}, \{N, Y\}, \{G, B\}, \{\underline{e}, \overline{e}\}) &= \mathbf{N} \end{aligned}$$

  - At this point one can observe the strategies of each player
    - **VC** can choose ONE pair of numbers in the real line as she cannot conditional on the state:[5]

$$[(w_{OK}, w_S)] \in [0, \infty)^2$$

    - **E** can choose for each offer whether to accept or not the contract and if accepted, then choose effort conditional on the state

$$\{(Y, e(G), e(B)), N\}$$

    - Player **N** plays mechanically in two different places:

$$1 - \Pr(B) = \eta = \Pr(G)$$

$$1 - \Pr(\underline{\pi}|\overline{e}, G) = \Pr(\overline{\pi}|\overline{e}, G) = p > q = \Pr(\overline{\pi}|\underline{e}, G) = 1 - \Pr(\underline{\pi}|\underline{e}, G)$$

$$\Pr(\underline{\pi}|\overline{e}, B) = \Pr(\underline{\pi}|\underline{e}, B) = 1 > 0 = \Pr(\overline{\pi}|\overline{e}, B) = \Pr(\overline{\pi}|\underline{e}, B)$$

- Information for each player (subhistories)

$$I_{\mathbf{VC}} = (\emptyset) \qquad I^1_{\mathbf{E}} = (\{w_{OK}, w_S\}) \qquad I^2_{\mathbf{E}} = (\{w_{OK}, w_S\},\ Y,\ \{G, B\})$$

[5] Again, salaries cannot be negative by assumption.

- The payoffs

$$U_j(s,\{w_{OK},w_S\},(Y,\underline{e}),\pi)=\begin{cases}\sqrt{w_{OK}}-\underline{C} & j=E,s\in\{G,B\},\ \pi\in\{\underline{\pi},\overline{\pi}\}\\ \underline{\pi}-w_{OK} & j=VC,s\in\{G,B\},\ \pi=\underline{\pi}\\ \overline{\pi}-w_{OK} & j=VC,s\in\{G,B\},\ \pi=\overline{\pi}\end{cases}$$

$$U_j(s,\{w_{OK},w_S\},(Y,\overline{e}),\pi)=\begin{cases}\sqrt{w_S}-\overline{C} & j=E,s\in\{G,B\},\ \pi\in\{\underline{\pi},\overline{\pi}\}\\ \underline{\pi}-w_S & j=VC,s\in\{G,B\},\ \pi=\underline{\pi}\\ \overline{\pi}-w_S & j=VC,s\in\{G,B\},\ \pi=\overline{\pi}\end{cases}$$

$$U_j(s,\{w_{OK},w_S\},N)=\begin{cases}\underline{U} & j=E,s\in\{G,B\},\overline{\pi}\}\\ 0 & j=VC,s\in\{G,B\},\ \pi=\underline{\pi}\end{cases}$$

where again we used $\{w_{OK},w_S\}$ to simbolize that this is a pair of two numbers in the real line.

We can also describe the game using a Game tree which we do in Figure 3

Again, we sidestep Nash equilibria and focus on Subgame Perfect equilibria assuming that $w_{OK}=w_S$.

- By backward induction we look at the last nodes:
  - After **E** observes that the state is B, they realize that there is no effect for exerting low or high effort in terms of the payoffs as the outcome is always low and she gets in both situations. It follows that the best response in this node is to exert low effort $\underline{e}$.
  - After **E** observes that the state is G, they realize that since $w_{OK}=w_S$, her salary is the same no matter the outcome. It follows that the best response in this node is to exert low effort $\underline{e}$ and save on effort costs.
  - It follows that the unique best response in any subgame Perfect nash equilibrium after having accepted the contract is to exert low effort in bot states
- Since they always exert low effort, and the compensation is the same after a good or bad outcome, $w_{OK}=w_S$, the payoffs for **E** are the same in both states and accepting the contract lead to payoffs $\sqrt{w_{OK}}-\underline{C}=\sqrt{w_S}-\underline{C}$
- The payoffs of both players for reaching the node after **E** accepts the contract are then

$$(\eta(q\overline{\pi}+(1-q)\underline{\pi})+(1-\eta)\underline{\pi}-w_{OK},\sqrt{w_{OK}}-\underline{C})$$

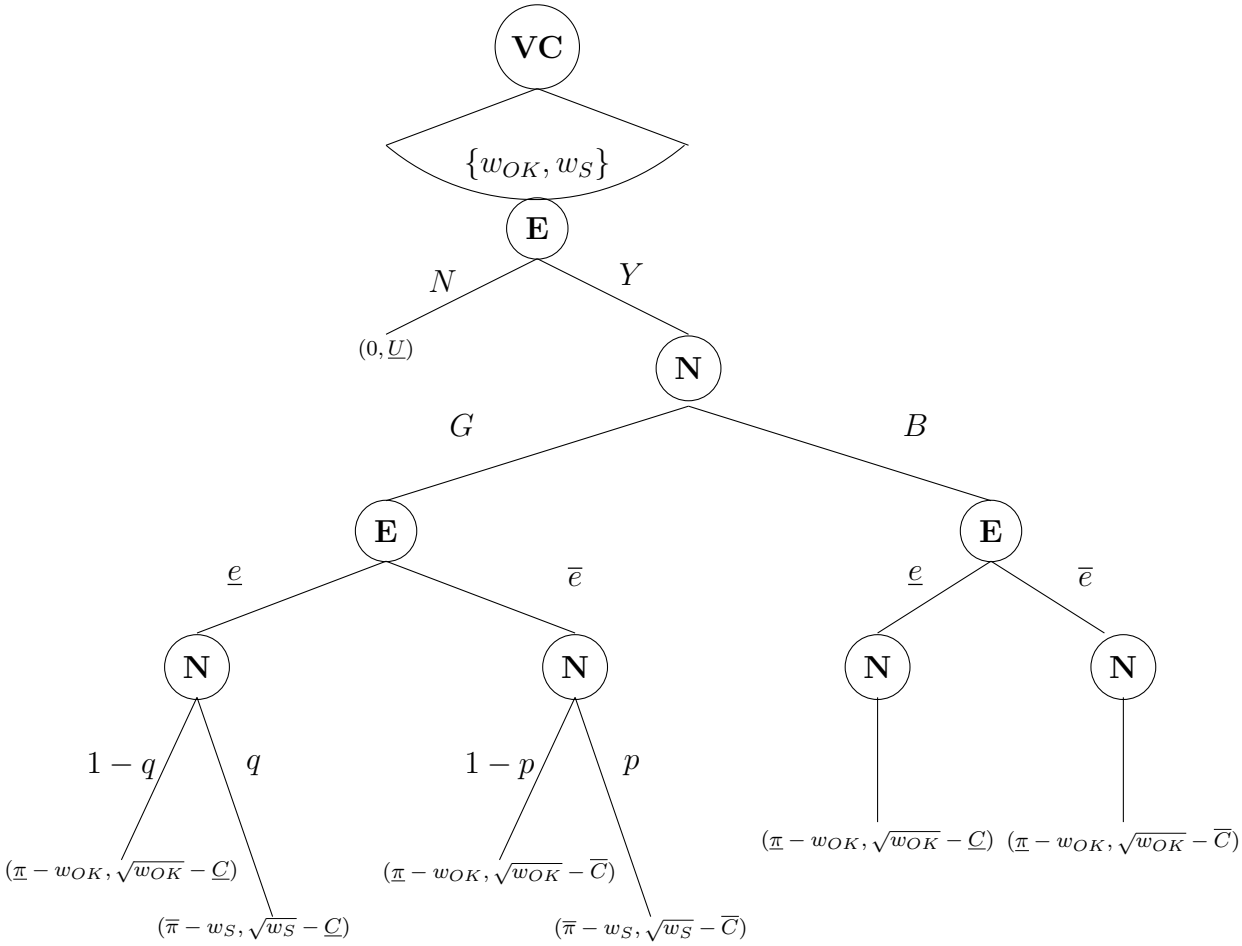

FIGURE 3. Game tree describing the game between the venture capitalist (**VC**) and the entrepreneur (**E**) with unobservable effort and state only observable after contracting by **E** (Moral Hazard). The arc signifies that there are multiple possible offers $\{w_{OK}, w_S\}$; in particular $\{w_{OK}, w_S\} \in [0, \infty)^2$. We have separated the decision of **E** in two different nodes for convenience only.

- The decision to accept or not the contract depends on whether the salaries compensate for outside option and the cost of effort. It follows that the unique best response in any subgame perfect Nash equilibrium for **E** in the node in which they need to accept or reject the contract is[6] to accept the contract if $\sqrt{w_{OK}} - \underline{C} \geq \underline{U}$ and to decline otherwise.
- Now for **VC** the decision is quite simple.
  - If the contract is such that $w_{OK} \geq (\underline{U} + \underline{C})^2$, **E** accepts the contract and exerts low effort in both states leading to **VC**'s payoffs

$$\eta(q\overline{\pi} + (1-q)\underline{\pi}) + (1-\eta)\underline{\pi} - (\underline{U} + \underline{C})^2$$

[6]Recall that this is not the best response function as we are only focusing on one node; this is the behavioral strategy conditional on optimal behavior in subsequent nodes.

– If **VC** offers any other contract, **E** rejects the offer leading to 0 payoffs for **VC**

The next proposition summarizes the above results

Proposition 3. *If*

$$\eta(q\overline{\pi} + (1-q)\underline{\pi}) + (1-\eta)\underline{\pi} \geq (\underline{U} + \underline{C})^2$$

The unique subgame perfect Nash equilibrium is

- ***VC*** *offers a contract*

$$w_{OK} = (\underline{U} + \underline{C})^2$$

- ***C*** *accepts any contract with*

$$w_{OK} \geq (\underline{U} + \underline{C})^2$$

  and rejects otherwise.

- *In both states* ***E*** *exerts low effort.*

3.2. **Debrief.** When **VC** cannot use performance-pay, **E** never exerts high effort as an employe but he would have done so if they were the owner of the firm and the state were G. This is an inefficient outcome as in an example of moral hazard: **VC** faces all the risk and **E** does not have any skin in the game leading to the lowest possible effort.

We will show that, in order to overcome the moral hazard problem, **VC** employs performance-based pay: performance pay allows **VC** to make **E** have skin in the game and to take the responsibility of reducing the risk. Indeed, in state G there is the possibility of reducing the risk by exerting high effort and raisng the probability of the good outcome to p instead of q By sharing risks and rewards or making outcomes more contingent on verifiable performance, these mechanisms reduce the agent's ability and temptation to exploit the hidden action, thereby steering outcomes closer to the principal's objectives.

4. Soving the full model

In the full game, **E** the entrepreneur's effort is not directly observable, meaning that wages cannot be structured based on actual effort. However, since the quality of the product is observable, the market can distinguish between a product of **OK** quality and one of **S** quality, leading to different profit outcomes in each case. As a result, **VC** can design a compensation scheme where the entrepreneur receives a wage of w_{OK} if the product is of **OK** quality and a different wage, w_S, if the product is of **S** quality.

The question is then, how should **VC** choose the salaries offered to the entrepreneur? It is a good idea to recap here

- We are considering a sequential game in which player **VC** (the venture capitalist) moves first by choosing the wage structure. Then, player **E** (the entrepreneur) decides whether to accept the offer and, if so, what level of effort to exert after observing the state. The resulting payoffs are stochastic, as they depend on an external factor, the so called player **N**, that determines the final outcome.
- **VC** aims to maximize the profits generated from the product's development by providing funds to **E**. In return, **E** allows **VC** to appropriate all profits. At first glance, this setup resembles a standard employer-employee relationship, but as we will see, it has unique characteristics that make it special.
- Because **VC** cannot directly observe **E**'s effort, she cannot condition wages on actual effort. Instead, she can only create *incentives* for **E** to exert the desired effort by conditioning the wage on the observable outcome. By carefully designing the wage structure, **VC** can *induce* **E** to choose the optimal effort level.
- Although **E** recognizes that, in an ideal world where effort is observable, the optimal strategy would be to exert high effort if they were the owner, they do not know whether such effort can still be induced by **VC** when effort is unobservable. Therefore, **VC** must first determine the cost of inducing different levels of effort. She must then choose the effort level that maximizes profits while accounting for these incentive costs. In other words, the cost of inducing effort arises indirectly from the wage structure rather than directly from **E**'s intrinsic effort cost.

As usual, we solve all possible subgames by backward induction and find the optimal responses of the players that need to move in those situations.

Note that in the last node **N** moves and the moves of this player are purely random. It follows that we can apply directly the notion of subgame perfect equilibrium if payoffs are represented in expectation for the other players. In particular note that for a given offer $\{w_{OK}, w_S\}$ the expected utility of **E** for each level of effort in state G is given by

$$EU_{\mathbf{E}}(\underline{e}|\{w_{OK}, w_S\}) = q\sqrt{w_S} + (1-q)\sqrt{w_{OK}} - \underline{C}$$

$$EU_{\mathbf{E}}(\overline{e}|\{w_{OK}, w_S\}) = p\sqrt{w_S} + (1-p)\sqrt{w_{OK}} - \overline{C}$$

On the other hand, for **VC** expected payoff for each new final node in state G are given by

$$EU_{\mathbf{VC}}(\underline{e}|\{w_{OK}, w_S\}) = q(\overline{\pi} - w_S) + (1-q)(\underline{\pi} - w_{OK})$$
$$EU_{\mathbf{VC}}(\overline{e}|\{w_{OK}, w_S\}) = p(\overline{\pi} - w_S) + (1-p)(\underline{\pi} - w_{OK})$$

We can also infer the optimal response of **E** after observing state B using the same arguments as above: since payoffs are the same for any level of effort, their payoffs are independent of effort except for the cost of effort, rendering no effort optimal.

We can then reduce the Game tree to the one in Figure 4

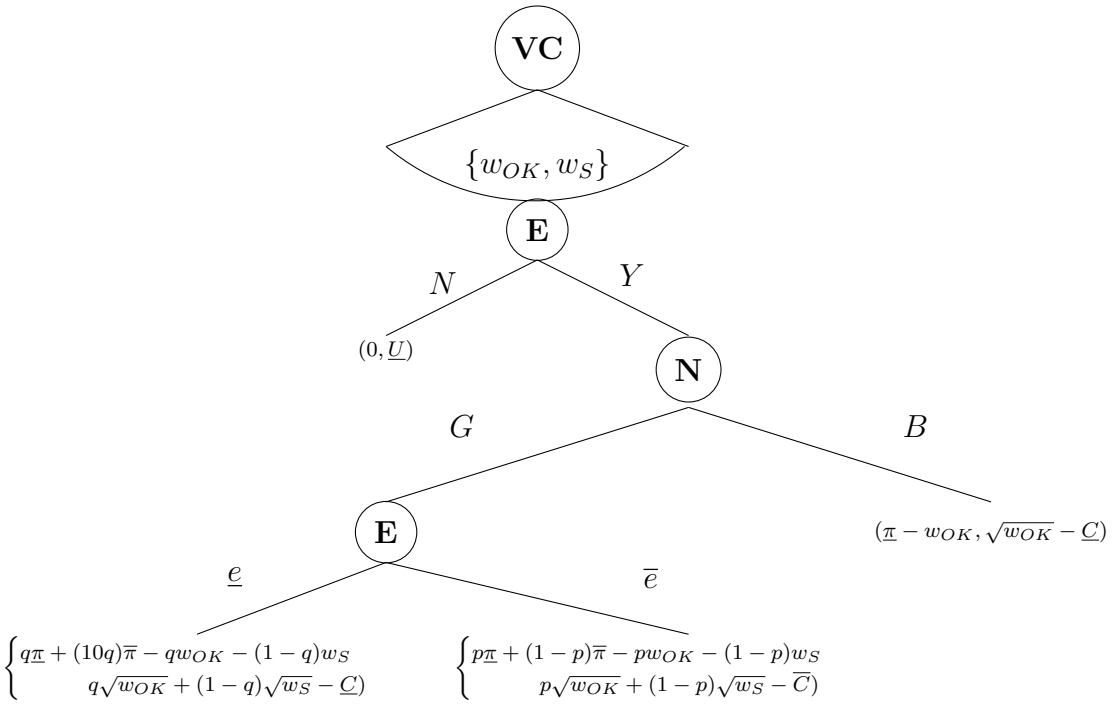

FIGURE 4. Game tree without **N** as a player in the final node.

We can continue to apply the backwards induction argument in the node in which **E** needs to choose effort after observing state G. **E** will exert high effort if

$$EU_{\mathbf{E}}(\overline{e}|\{w_{OK}, w_S\}) \geq EU_{\mathbf{E}}(\underline{e}|\{w_{OK}, w_S\})$$
$$p(\sqrt{w_S} - \sqrt{w_{OK}}) - \overline{C} \geq q(\sqrt{w_S} - \sqrt{w_{OK}}) - \underline{C}$$

is equivalent to

$$\boxed{(p-q)(\sqrt{w_S} - \sqrt{w_{OK}}) \geq \overline{C} - \underline{C}} \tag{4}$$

This is quite intuitive:

- On the right hand side of the inequality, $\overline{C} - \underline{C}$ is the marginal cost for making high effort instead of low effort.
- On the left hand side, is the expected marginal benefit of increasing effort. This is because $p - q$ is the difference between the good result with high effort and low effort, and $(\sqrt{w_S} - \sqrt{w_{OK}})$ is the total marginal benefit (in utility terms) for the high outcome.

If condition (4) holds we have that **E** exerts high effort in state G and low effort in state B leading to expected payoffs for both players in the node after accepting the contract as

$$\begin{aligned}(&\eta(p(\overline{\pi} - w_S) + (1-p)(\underline{\pi} - w_{OK}) + (1-\eta)(\underline{\pi} - w_{OK}) \\ &, \eta(p\sqrt{w_S} + (1-p)\sqrt{w_{OK}} - \overline{C}) + (1-\eta)(\sqrt{w_{OK}} - \underline{C})\big)\end{aligned}$$

or after some algebra

$$\big(\eta p(\overline{\pi} - w_S) + (1-\eta p)(\underline{\pi} - w_{OK}),\ \eta p\sqrt{w_S} + (1-\eta p)\sqrt{w_{OK}} - \eta\overline{C} - (1-\eta)\underline{C})\big)$$

Note that **E** will accept a contract that induces high effort in state G and low effort in state B, if the expected payoffs is higher than the outside option or if

$$\underline{U} \leq EU_{\mathbf{E}}(Y \mid \{w_{OK}, w_S\})$$

which is equivalent to

$$\boxed{\underline{U} \leq \eta p\sqrt{w_S} + (1-\eta p)\sqrt{w_{OK}} - \eta\overline{C} - (1-\eta)\underline{C}} \tag{5}$$

Before **VC** can decide what to do she needs to compare the benefits of inducing high effort in state G or accepting that in both states, **E** will exert low effort. The optimal way for **VC** to induce the right type of effort in each state, **VC** needs to calculate the wage structure that verifies (4) and (5).

4.0.1. *Inducing high effort in state G.* Let's consider now the profits for making **E** to choose high effort in state G only. In this case, the expected payoff is given by

$$\eta p(\overline{\pi} - w_S) + (1-\eta p)(\underline{\pi} - w_{OK}) = \underbrace{\eta p\overline{\pi} + (1-\eta p)\underline{\pi}}_{\text{Revenue}} - \underbrace{(\eta p w_S + (1-\eta p)w_{OK})}_{\text{Cost}}$$

In order for **E** to choose high effort in state G, it must be that (4) holds and for **E** to acceot the contract it must be that (5) holds. Note that **VC** chooses the wage structure

$\{w_{OK}, w_S\}$ and, conditional on **E** choosing high effort, the choice of **VC** does not affect revenue, the problem actually is to minimize the costs:

$$\begin{aligned}\min_{\{w_{OK},w_S\}\in[0,\infty)^2} \quad & \eta p w_S + (1-\eta p) w_{OK} \\ \text{subject to } & (p-q)(\sqrt{w_S} - \sqrt{w_{OK}}) \geq \overline{C} - \underline{C} \\ & \eta p \sqrt{w_S} + (1-\eta p)\sqrt{w_{OK}} \geq \underline{U} + \eta \overline{C} + (1-\eta)\underline{C}\end{aligned}$$

First of all, note that a fixed wage structure here will not work because the first constraint implies that

$$\sqrt{w_S} \geq \frac{\overline{C} - \underline{C}}{p-q} + \sqrt{w_{OK}} > \sqrt{w_{OK}}$$

will not hold. So it has to be that:

Proposition 1. *If **VC** wants to induce high effort, then it must be that the wage structure does depend on the quality of the product and it pays more when the quality is S, than when the quality is OK:*

$$w_S^* > w_{OK}^*$$

Assume that an optimal choice w_S^* and $w_{OK}^*$is such that the second inequality holds strictly, i.e.

$$\begin{aligned}(p-q)(\sqrt{w_S^*} - \sqrt{w_{OK}^*}) &\geq \overline{C} - \underline{C} \\ \sqrt{w_S^*} + (1-\eta p)\sqrt{w_{OK}^*} &> \underline{U} + \eta \overline{C} + (1-\eta)\underline{C}\end{aligned}$$

Note that both salaries could be reduced by the same small amount, this will not affect the first inequality, reduce the total costs, and still verify the second inequality. It follows then that the second inequality must be tight and the problem reduces to

$$\begin{aligned}\min_{\{w_{OK},w_S\}\in[0,\infty)^2} \quad & \eta p w_S + (1-\eta p) w_{OK} \\ \text{subject to } & (\sqrt{w_S} - \sqrt{w_{OK}}) \geq \frac{\overline{C} - \underline{C}}{p-q} \\ & \eta p \sqrt{w_S} + (1-\eta p)\sqrt{w_{OK}} = \underline{U} + \eta \overline{C} + (1-\eta)\underline{C}\end{aligned}$$

This problem can be easily solved by using Lagrange optimization.[7] In fact, it is easier if we think about it a little:

[7]The objective is linear and the first two constraints are quasiconcave on $w_{OK} < w_S$

(1) Note that the second line could be reduced by noting that

$$w_S = \left(\frac{\underline{U} + \eta\overline{C} + (1-\eta)\underline{C}}{\eta p} - \frac{1-\eta p}{\eta p}\sqrt{w_{OK}} \right)^2$$

(2) Using the above equation we have that the constraint that $w_S \geq 0$ is equivalent to

$$\frac{\underline{U} + \eta\overline{C} + (1-\eta)\underline{C}}{1-\eta p} \geq \sqrt{w_{OK}}$$

(3) Using the expression for w_S above and replacing in the reduced problem we have that the new problem for **VC** is only in one dimension

$$\begin{aligned} \min_{w_{OK} \geq 0} \quad & \eta p \left(\frac{\underline{U} + \eta\overline{C} + (1-\eta)\underline{C}}{\eta p} - \frac{1-\eta p}{\eta p}\sqrt{w_{OK}} \right)^2 + (1-\eta p) w_{OK} \\ \text{subject to } & \left(\left(\frac{\underline{U} + \eta\overline{C} + (1-\eta)\underline{C}}{\eta p} - \frac{1-\eta p}{\eta p}\sqrt{w_{OK}} \right) - \sqrt{w_{OK}} \right) \geq \frac{\overline{C} - \underline{C}}{p-q} \\ & \frac{\underline{U} + \eta\overline{C} + (1-\eta)\underline{C}}{1-\eta p} \geq \sqrt{w_{OK}} \end{aligned}$$

and rearranging, this reduces to

$$\begin{aligned} \min_{w_{OK} \geq 0} \quad & \eta p \left(\frac{\underline{U} + \eta\overline{C} + (1-\eta)\underline{C}}{\eta p} - \frac{1-\eta p}{\eta p}\sqrt{w_{OK}} \right)^2 + (1-\eta p) w_{OK} \\ \text{subject to } & \underline{U} + \underline{C} + \eta(\overline{C} - \underline{C}) - \eta p \frac{\overline{C} - \underline{C}}{p-q} \geq \sqrt{w_{OK}} \\ & \frac{\underline{U} + \underline{C} + \eta(\overline{C} - \underline{C})}{1-\eta p} \geq \sqrt{w_{OK}} \end{aligned}$$

(4) Note that

$$\frac{\underline{U} + \underline{C} + \eta(\overline{C} - \underline{C})}{1-\eta p} > \underline{U} + \underline{C} + \eta(\overline{C} - \underline{C}) - \eta p \frac{\overline{C} - \underline{C}}{p-q}$$

so the second condition holds if the first one holds.

(5) This reduces the problem to, where we used some algebra in the second condition,

$$\begin{aligned} \min_{w_{OK} \geq 0} \quad & (\eta p)^{-1} \left(\underline{U} + \underline{C} + \eta(\overline{C} - \underline{C}) - (1-\eta p)\sqrt{w_{OK}} \right)^2 + (1-\eta p) w_{OK} \\ \text{subject to } & \underline{U} + \underline{C} + \eta(\overline{C} - \underline{C}) - \eta p \frac{\overline{C} - \underline{C}}{p-q} \geq \sqrt{w_{OK}} \end{aligned}$$

(6) Note that the objective function can be written as

$$\begin{aligned} & (\eta p)^{-1} \left(\underline{U} + \underline{C} + \eta(\overline{C} - \underline{C}) - (1-\eta p)\sqrt{w_{OK}} \right)^2 + (1-\eta p) w_{OK} \\ & = \frac{\left(\underline{U} + \underline{C} + \eta(\overline{C} - \underline{C}) \right)^2}{\eta p} + \frac{1-\eta p}{\eta p} \left(w_{OK} - 2\left(\underline{U} + \underline{C} + \eta(\overline{C} - \underline{C}) \right) \sqrt{w_{OK}} \right) \end{aligned}$$

with derivative with respect to w_{OK} given by

$$\frac{1-\eta p}{\eta p}\left(\sqrt{w_{OK}}-\left(\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})\right)\right)\frac{1}{\sqrt{w_{OK}}}$$

(7) Note that given the constraint that the problem needs to verify

$$\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})-\eta p\frac{\overline{C}-\underline{C}}{p-q}\geq\sqrt{w_{OK}}$$

this expression is negative.

(8) It follows that increasing w_{OK} reduces the objective function and hence, the unique solution to the problem above must make this wage as high as possible, which implies that the constraint must hold with equality

$$\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})-\eta p\frac{\overline{C}-\underline{C}}{p-q}=\sqrt{w_{OK}}$$

Proposition 2. *If **VC** wants to induce high effort, then she chooses a wage structure that solves*

$$\begin{aligned}\sqrt{w_{OK}^*}&=\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})-\eta p\frac{\overline{C}-\underline{C}}{p-q}\\&=\underline{U}+\underline{C}-\eta(\overline{C}-\underline{C})\frac{q}{p-q}\\\sqrt{w_S^*}&=\frac{\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})}{\eta p}-\frac{1-\eta p}{\eta p}\sqrt{w_{OK}^*}\\&=\underline{U}+\underline{C}+\frac{1-\eta q}{p-q}(\overline{C}-\underline{C})\end{aligned}$$

and obtain the expected profits

$$E(\pi|w_S^*,w_{OK}^*)=\eta p(\overline{\pi}-w_S^*)+(1-\eta p)(\underline{\pi}-w_{OK}^*)$$

Note that

$$\begin{aligned}\sqrt{w_S^*}&=\sqrt{w_{OK}^*}+\frac{\overline{C}-\underline{C}}{p-q}\\w_S^*&=w_{OK}^*+2\frac{\overline{C}-\underline{C}}{p-q}\sqrt{w_{OK}^*}+\left(\frac{\overline{C}-\underline{C}}{p-q}\right)^2\end{aligned}$$

which implies

$$\begin{aligned}E(\pi|w_S^*,w_{OK}^*)&=\eta p\overline{\pi}+(1-\eta p)\underline{\pi}-\left(\sqrt{w_{OK}^*}+\eta p\frac{\overline{C}-\underline{C}}{p-q}\right)^2+(1-\eta p)\,\eta p\left(\frac{\overline{C}-\underline{C}}{p-q}\right)^2\\&=\eta p\overline{\pi}+(1-\eta p)\underline{\pi}-\left(\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})\right)^2+(1-\eta p)\,\eta p\left(\frac{\overline{C}-\underline{C}}{p-q}\right)^2\end{aligned}$$

4.1. **Optimal choice of VC.** The first mover, **VC**, understands that the wage structure that she proposes, $\{w_{OK}, w_S\}$ will induce **E** to either accept the wage offer and choose high effort, $\overline{e}$, in state G. She can also choose to induce low effort in both states which is obtained in the preceding section (see Proposition 3) and yields

$$E(\pi|w^*) = \eta q\overline{\pi} + (1-\eta q)\underline{\pi} - (\underline{U}+\underline{C})^2$$

The question is then whether $E(\pi|w^*)$ is higher than $E(\pi|w_S^*, w_{OK}^*)$ or the other way round. Note that

$$E(\pi|w_S^*, w_{OK}^*) \geq E(\pi|w^*)$$

if and only if

$$\begin{aligned}
\eta p\overline{\pi} + (1-\eta p)\underline{\pi} - \left(\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})\right)^2 + (1-\eta p)\,\eta p\left(\frac{\overline{C}-\underline{C}}{p-q}\right)^2 &\geq \eta q\overline{\pi} + (1-\eta q)\underline{\pi} - (\underline{U}+\underline{C})^2 \\
\eta(p-q)\,(\overline{\pi}-\underline{\pi}) + (1-\eta p)\,\eta p\left(\frac{\overline{C}-\underline{C}}{p-q}\right)^2 &\geq \left(\underline{U}+\underline{C}+\eta(\overline{C}-\underline{C})\right)^2 - (\underline{U}+\underline{C})^2 \\
\eta(p-q)\,(\overline{\pi}-\underline{\pi}) + (1-\eta p)\,\eta p\left(\frac{\overline{C}-\underline{C}}{p-q}\right)^2 &\geq \eta(\overline{C}-\underline{C})\left(2\,(\underline{U}+\underline{C}) + \eta(\overline{C}-\underline{C})\right) \\
(p-q)\frac{\overline{\pi}-\underline{\pi}}{\overline{C}-\underline{C}} + (1-\eta p)\,\frac{p}{p-q}\frac{\overline{C}-\underline{C}}{p-q} &\geq 2\,(\underline{U}+\underline{C}) + \eta(\overline{C}-\underline{C})
\end{aligned}$$

which is equivalent to

$$\boxed{(p-q)\frac{\overline{\pi}-\underline{\pi}}{(\overline{C}-\underline{C})^2} - 2\frac{\underline{U}+\underline{C}}{\overline{C}-\underline{C}} \geq \eta - \frac{(1-\eta p)p}{(p-q)^2}} \tag{6}$$

The next proposition summarizes the analysis

Proposition 4. *If (6) holds, then the unique subgame perfect equilibrium i scharacterized by*

- ***VC** offers the wages*

$$\begin{aligned}
w_{OK}^* &= \left(\underline{U}+\underline{C} - \frac{\eta q}{p-q}(\overline{C}-\underline{C})\right)^2 \\
&< w_S^* = \left(\underline{U}+\underline{C} + \frac{1-\eta q}{p-q}(\overline{C}-\underline{C})\right)^2
\end{aligned}$$

- ***E** accepts any contract that verifies*

$$\begin{aligned}
(\sqrt{w_S} - \sqrt{w_{OK}}) &\geq \frac{\overline{C}-\underline{C}}{p-q} \\
\eta p\sqrt{w_S} + (1-\eta p)\sqrt{w_{OK}} &\geq \underline{U} + \eta\overline{C} + (1-\eta)\underline{C}
\end{aligned}$$

and rejects otherwise

- *If the contract is accepted* ***E*** *exerts high effort in state G and low effort in state B if*

$$(\sqrt{w_S} - \sqrt{w_{OK}}) \geq \frac{\overline{C} - \underline{C}}{p - q}$$

and otherwise exerts low effort in both states

If (6) *does not hold, there are multiple equilibria and all equilibria are payoff equivalent to the following contract*

- ***VC*** *offers the wages*

$$w^*_{OK} = w^*_S = (\underline{U} + \underline{C})^2$$

- ***E*** *accepts any contract that verifies*

$$\min\{w_S, w_{OK}\} \geq (\underline{U} + \underline{C})^2$$

and rejects otherwise

- *If the contract is accepted,* ***E*** *exerts high effort in state G and low effort in state B if*

$$(\sqrt{w_S} - \sqrt{w_{OK}}) \geq \frac{\overline{C} - \underline{C}}{p - q}$$

and otherwise exerts low effort in both states

uses the same strategies to accept or reject the contract as above but ***VC*** *offers and after accepting the contract*

5. Takeaways

5.1. **The wage structure.** When effort is not observable, inducing high effort requires to generate the incentives for **E** to choose high effort endogenously. These incentives are reflected in the fact that $w^*_{OK} < w^*_S$ and **E** receives a bonus if things go well. Although there is a bonus, **E**'s wage when the product is only of OK quality is lower than under complete information, and if the product is of superb quality **E**'s wage is higher than under complete information.

Nevertheless, in expectation **E** receives the same utility as in the case of complete information. Algebra is not needed for this result as the wage contract in expectation verifies (5) which leaves the agent indifferent between taking the contract and taking the outsid eoption.

5.2. **The costs of asymmetric information.** There are two aspects of asymmetric information that are important to highlight. First, when high effort is implemented, **VC** makes less profits than in the case of complete information. This is because of the incentives that need to be generated and are costly to generate. The cost is given by the fact that **E** is risk averse and in order to compensate him properly, there is the need to pay excessively in expectation so the loss is "transferred" to **VC**. In that sense, the incentives require to generate more risk than necessary although the player that bear those costs is the one that is best prepared for it as it is also the uninformed party.

Since the incentives are costly, and although it is efficient to implement high effort, as expressed by assumption 2, it is not always the case that high effort can be implemented. This is because, **VC** must create the incentives for **E** to exert high effort and, sometimes, there is not enough upside (when condition (6) does not hold). In these case, low effort is implemented with a structure of fixed wages which is inefficient

5.3. **The impact of better projects.** Note that there are two components to determine better projects: the high return $\overline{\pi}$, the low return $\underline{\pi}$, and the probabilities of the high return p and q.

5.3.1. *The effect of higher high return:* The optimal wages do not depend on $\overline{\pi}$ as they are only determined by compensating for the outside option $\underline{U}$ and cost of effort $\overline{C}$. This means that higher $\overline{\pi}$ only affects the profits of **VC** and, indirectly, whether high effort can be exerted or not. Indeeed, note that the condition for high effort to be an equilibrium outcome is easier to verify if $\overline{\pi}$ increases.

5.3.2. *The effect of higher low return:* The effect of $\underline{\pi}$ is quite sinister. Again, **E** is not affected by its change, except that a higher $\underline{\pi}$ makes it more difficult to induce high effort and, hence, may change the salary structure into a low effort salary structure. Indeed, when $\underline{\pi}$ increases, the expected profit of **VC** also increases, but when $\underline{\pi}$ increases, it also makes inducing low effort a relatively decent option as the difference $\overline{\pi} - \underline{\pi}$ decreases. Since inducing high effort is still costly, and its cost does not change with the returns of the project, at some point **VC** is unwilling to pay for the incentives.

5.3.3. *The effect of safer projects.* When p increases, it is more likely that the product is of superb quality. This means that the outcome in which **E** receives a bonus and **VC** makes the most profits is more likely. Because, of this increased probability, **E** is more willing to accept a lower pay (intensive margin) for a more likely outcome (extensive

margin) which reduces the actual cost of incentives while /textbfVC is willing to take on "more" risk. Indeed, w^*_{OK} increases with p while w^*_S decreases and reduces the bonus that **E** receives (a lower bonus that occurs more often).

Clearly, a more likely good outcome and lower bonus, increases the marginal benefit of funding the project in two ways and it is easier to implement high effort.

Reality check: This gives us a nice prediction about wage structures in different industries. Industries with highly risky projects, where exerting effort is crucial, tend to rely more on bonuses than in fix salaries. This can be seen in the wage structure of financial services and banking services: the former are bonus driven and the latter are salary driven.

5.3.4. *A word of caution.* The fact that the returns of the project do not affect at all the salary structure but only the feasibility of high effort, is due to three important assumptions. First, **VC** has linear preferences (risk neutral) and **E** has linear square root preferences (risk averse). If **VC** also has preferences with some level of risk aversion, for example $x^{1/3}$ then $\overline{\pi}$ and $\underline{\pi}$ would also affect the wage structure. Second, we have assume a particular sequential structure and a particular contract. It could make sense in this et up for **VC** and **E** to communicate half way through (for example, when **E** sees the actual state). More advance game theory is needed for different sequential structures, contracts and communication, and we will not look at it here. Third, there is no competition for the services of **E**. If there were many different venture capitalists, the wage structure would reflect the competition between them and how much each of them can pay **E**. Again, this requires more advanced Game Theory.